Effects of changing orders in the update rules on traffic flow

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Based on the Nagel-Schreckenberg (NaSch) model of traffic flow, we study the effects of the orders of the evolutive rule on traffic flow. It has been found from simulation that the cellular automaton (CA) traffic model is very sensitively dependent on the orders of the evolutive rule. Changing the evolutive steps of the NaSch model will result in two modified models, called the SDNaSch model and the noise-first model, with different fundamental diagrams and jamming states. We analyze the mechanism of these two different traffic models and corresponding traffic behaviors in detail and compare the two modified model with the NaSch model. It is concluded that the order arrangement of the stochastic delay and deterministic deceleration indeed has remarkable effects on traffic flow.

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I. Introduction

Recently, traffic problems have attracted considerable attention from scientists because of the observed nonequilibrium phase transitions and various nonlinear dynamical phenomena [1]. Various traffic models, including the cellular automaton models, the car-following models, the hydrodynamic models, and the gas kinetic models have been developed [2], and numerical empirical observations have been reported [3]. Recent measurements have shown that the flow-density relation in the fundamental diagram is rather complicated [3]. Analysis of measured data in highway traffic indicated that the flow is not a unique function of the density in some situations. A different scenario for jam formation was proposed in 1994 by Kerner and Konhäuser at DaimlerChrysler [4]. For a better understanding of such complex traffic phenomena and reproducing the empirical data, a very simple cellular automaton model for single-lane traffic was presented by Nagel and Schreckenberg in 1992 [5,6], called the NaSch model. The model is able to reproduce the basic phenomena of real traffic, such as the spontaneous formation of jams, by using very simple rules. The state of the system at the time $t+1$ could be obtained from the state at the time t by applying the following rules to all cars at the same time:

(i) Acceleration,

$$v_n \to \min(v_n + 1, v_{\max}).$$

(ii) Deterministic deceleration to avoid accidents,

$$v_n \to \min(\mathrm{v}_n, \mathrm{gap}_n).$$

(iii) Randomization,

$$v_n \to \max(\mathrm{v}_n - 1, 0) \text{ with probability } p.$$

(iv) Update of positions,

$$x_n(\mathrm{t}+1) \to \mathrm{x}_n + v_n.$$

The NaSch model has been recognized as the pioneering work for simulating real traffic flow with cellular automaton (CA) models. Its dynamics is formulated as follows [7]:

$$x_i(\mathrm{t}+1) = x_i(t) + \max[0 \ \min\{v_{\max}, x_{j+1} - x_j - 1, x_j(t) - x_j(t-1) + 1\} - \xi_j(t)], \quad (1)$$

where the Boolean random variable $\xi_j(t)=1$ with probability p and 0 with probability $1-p$. The vehicles are updated in parallel according to the NaSch rules: motion, acceleration, deceleration, and randomization. The most important empirically measured quantities in traffic flow are usually shown in the fundamental diagram, which reflects the transit capacity for a one-lane traffic-flow model. However, the transit capacity given by the NaSch model was underestimated compared with the actual one in real traffic. The maximum flow (i.e., the road capacity) obtained by numerical simulation with the NaSch model is much lower than 2500 vehicles/($h*$ lane) given by measurements in highway traffic [8]. Moreover, the metastable state with two branches in the fundamental diagram has not been given by the NaSch model. To improve the situation, a variety of modifications to the NaSch model have been proposed by introducing the slow-to-start rules, among which are the VDR model [9], the T^2 model [10], and the BJH model [11]. They are able to reproduce metastable states and exhibit a clear separation of the congestion and free-flow regions in a space-time plot. The fundamental diagram obtained by numerical simulation shows the road capacity approaches to the observed data more closely. Although the NaSch model is a minimum model in the sense that all four steps are necessary to reproduce the basic features of real traffic, the only change in the order of the evolutive steps will result in many different traffic models. Nevertheless, from Eq. (1), it is not clear how to reflect the steps of evolution. In this paper, we attempt to study the effects of the change in the evolutive steps on traffic flow. We analyze the mechanisms of two different traffic models caused by

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changing the update steps in detail, which correspond to the traffic behaviors, and compare them with those deduced from the NaSch model. We find that the CA traffic model is very sensitively dependent on the orders of evolutive rule. Modification of the evolutive steps will result in different fundamental diagrams and jamming states with different properties as a counterpart to the different real traffic. Some modification leads to quite satisfactory results in numerical simulation in comparison with the empirical data, and the order of randomization deceleration indeed has a great effect on the description of the metastable states and separation phenomena. Moreover, the primary jamming state in the NaSch model might disappear by modifying of the evolutive steps, and the basic characteristics of synchronization flow will occur instead.

II. Description of the Model and Discussion

A. SDNaSch model and analysis

In the NaSch model, step (i) reflects the general tendency of the drivers to drive as fast as possible, if they are allowed to do so, within the maximum speed limit. Step (ii) is intended to avoid collision between the successive vehicles. The randomization in step (iii) actually includes the influences of the noise on the whole procedure of evolution. It has combined three different behavioral patterns into one computational rule [12]: fluctuations at maximum speed, retarded acceleration, and overreactions at braking. This mainly takes into account the different behavioral patterns of the individual drivers especially, nondeterministic acceleration as well as overreaction in the slowing-down process, which is crucially important for the spontaneous formation of traffic jams. Even changing the precise order of the steps of the update rules in the NaSch model would change the properties of the model. Some authors claimed that after exchanging the order of steps (ii) and (iii), there will be no overreactions at braking and thus no spontaneous formation of jams [2,13]. However, we attempt to examine this problem by numerical simulation by exchanging the order of steps (ii) and (iii), and find that this rearrangement of the update rules will possibly lead to the spontaneous formation of jams. The update rules of our model are as follows:

(i) Acceleration,

$$v_n \rightarrow \min(\mathrm{v}_n + 1, v_{\max}).$$

(ii) Randomization,

$$v_n \rightarrow \max(\mathrm{v}_n - 1, 0) \text{ with probability } p.$$

The noise affects on the process of deterministic acceleration. Thus, vehicles do not have to decelerate further.

(iii) Deterministic deceleration to avoid accidents,

$$v_n \rightarrow \min(\mathrm{v}_n, \mathrm{gap}_n).$$

(iv) Update of positions,

$$x_n(\mathrm{t}+1) \rightarrow \mathrm{x}_n + v_n.$$

In simulation with the above update rules, we represent a lane using a one-dimensional lattice of L cells with periodic boundary conditions, Each cell is either empty or occupied by just one vehicle with discrete velocity v. Velocity ranged from 0 to $v_{\max}=5$, where $v_{\max}$ is the speed limit. We use the same maximum velocity for all vehicles and consider one type of vehicle moving only along one direction. We simulate a system of length $L=5\times 10^3$, which corresponds to the length of the actual road around 37.5 km. One time step Δt is 1 s, which is of the order of the reaction time for humans. Then, the maximum velocity $v_{\max}=5$ corresponds to 135 km/h in real traffic. Let v_n and x_n denote the current velocity and position of the nth vehicle, respectively. We denote $\mathrm{gap}_n(\mathrm{t})$ by $\mathrm{gap}_n(\mathrm{t})=\mathrm{x}_{n+1}-\mathrm{x}_n-1$, which is the number of empty cells in front of the nth vehicle. The computational formulas are given as follows.

Average density,

$$\rho = N/L. \tag{2}$$

Mean velocity,

$$V=\sum_{i=1}^{n} v_i(t)/N=\sum_{t=t_0}^{T+t_0-1} v_i(t)/T. \tag{3}$$

Flow,

$$J=\rho V. \tag{4}$$

The numerical simulation was performed according to the above rules. For each simulation, we chose the probability $p=0.25$. Each run is first conducted for 5×10^4 time steps in order to remove the transient effects and then the data are recorded in successive 5×10^4 time steps. The fundamental diagram is obtained by averaging over 50 runs of simulations.

At the initial instant, N vehicles are uniformly distributed on the lane around the complete loop with an initial velocity 0. We obtain a fundamental diagram in our model with the same simulation conditions as those of the NaSch model and in the case of the various different delay probability, as shown in Figs. 1(a) and 1(b). Figure 1(a) indicates that our model leads to a higher value of maximum flow than that obtained with the NaSch model by 40%, which is close to the observed data (2500 vehicles/h* lane) [8]. In fact, when a driver finds the dense vehicles in the front on the road, he will first delay at random and estimate whether he should make vehicles decelerate or not by observing and evaluating his anticipation velocity and headway between successive vehicles. If he finds his anticipation velocity will surpass headway, he brakes agilely. Because the randomization is first taken into account, braking times in the state of free flow will be reduced and more vehicles with the maximum velocity will cause the increase of capacity, while vehicles cannot keep the maximum velocity at the dense density and the fluctuation of velocity leads to the spontaneous formation of jams and capacity drops. In contrast to the NaSch model, it makes more vehicles keep higher or even maximum velocity. This model is thus called the sensitive drive model or the SDNaSch model.

We can capture jamming of vehicles through the space-time plot Fig. 2(a) to describe the evolution of traffic flow and further illustrate the effects of the spontaneous formation

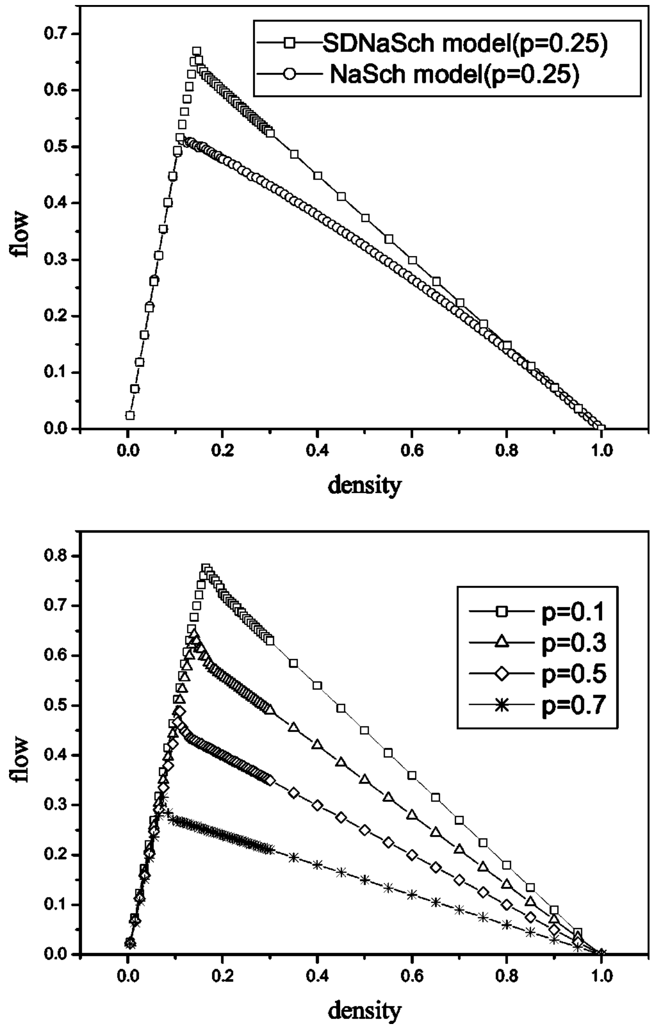

FIG. 1. (a) Fundamental diagram via numerical simulation with the same condition ($v_{\max}=5$), $L=5\times10^3$. Obviously, the flow of the new model is higher than that of the NaSch model. (b) Fundamental diagram via numerical simulation with the same condition ($v_{\max}=5$, $L=5\times10^3$) under a different delay probability.

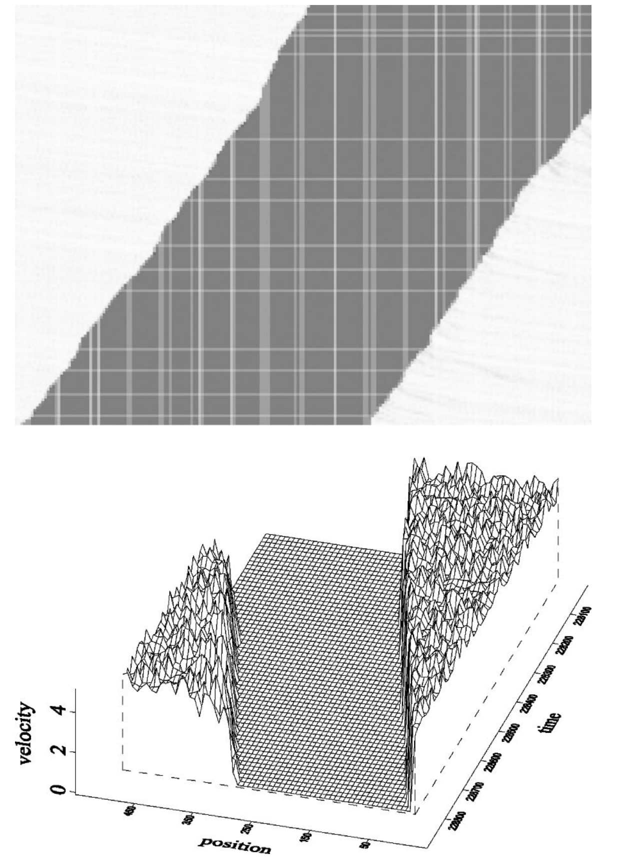

FIG. 2. (a) The space-time plot describes the evolution of traffic flow. The plot apparently shows the the spontaneous formation of traffic jams. The dark regions represent where the static vehicles collect to form the jam and the gray regions are free flow. The jam is evidently separated by the free flow, i.e., called separation of phase. (The horizontal direction is space in 500 cells and the vertical downward direction is increasing in time between 2.84×10^4 and 2.88×10^4 after removing the transient effects.) (b) is a plot of the velocity distribution corresponding to (a). It clearly shows that the velocity of collective vehicles approaches zero to form a jam ($\rho=0.2$).

of traffic jams. Figure 2(a) clearly shows the gray regions which correspond to free flow while the vehicle density is less than 0.2, and the dark regions which represent the static vehicles that cluster to form the jam. Some vehicles cannot keep their desired velocity and frequently decelerate at random, and fluctuations of velocity will cause some vehicles to stop, thereby forming a jam. The free flows are evidently separated by jams. This phenomenon is called “separation of phase,” which shows the possible existence of the metastable state for a long lifetime [9]. The separation of phase makes the flow slow down and the capacity drop. Figure 2(b) is a plot of the velocity distribution corresponding to Fig. 2(a). This figure clearly shows that the velocity of collective vehicles approaches zero to form jams.

Daganzo *et al.* have studied the effects of the driver behaviors on traffic flow [14], for instance with different branches for accelerating or decelerating traffic or different branches for distinct classes of drivers, e.g., rabbits and slugs. They have also found that the flow-density relation of the rabbits has a special reversed lambda shape, which was explained by assuming a collective loss of motivation of drivers to follow their predecessor closely, and the lane-specific evolution of the data points with time, including the “hysteresis” phenomenon and the lane-specific patterns in time series of speed (and flow) in both queued and unqueued traffic flow.

In order to take the hysteresis into account in our model, we conducted the numerical simulation under two different initial conditions [2,9]. One is the homogeneous distribution with the same headway, and the other is the megajam consisting of one large compact cluster of standing vehicles. Thus we obtain the fundamental diagram with two branches as shown in Fig. 3, which is similar to the results obtained with the VDR model. The results of the VDR model were originated from introducing two delay probabilities dependent on velocity instead of the constant randomization in the NaSch model, while the same result in our model comes from interchanging the order of the deterministic decelera-

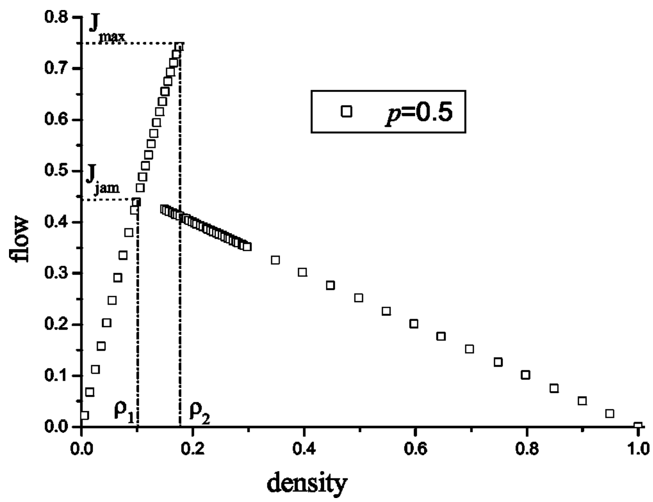

FIG. 3. Fundamental diagram via numerical simulation with two different initial conditions: uniform distribution state and inhomogeneous congestion ($v_{\max}$=5, L=5×10^3, p=0.5). The metastable state appears between $\rho_1 < \rho < \rho_2$.

tion and the stochastic one in the steps of the evolution rules. When density is near the range $\rho_1 < \rho < \rho_2$, the flow is discontinuous. The upper branch over the flow J_{jam} corresponds to the homogeneous traffic flow, which has larger flow with no jam due to the reduction of braking times in the sensitive driving. This case belongs to the metastable state and the flow reaches the maximum as $\rho \approx 0.18$. The lower branch corresponds to the traffic jam; the flow reduces rapidly because of the increase of the braking probability. It is obvious that there exists a hysteresis loop in the fundamental diagram. From the simulated results, we can get the following relations. In the regime of the upper branch as $0 < \rho < \rho_2$, every vehicle can move with the free-flow velocity $v_f = (1-p)v_{\max} + p(v_{\max}-1) = v_{\max} - p$, therefore the flow is given by

$$J_f = \rho(v_{\max} - p) = \rho v_f. \tag{5}$$

In the regime of the lower branch as $\rho_2 < \rho$, the average waiting time T_w of the first vehicles at the head of the megajam is given by the minimum value $T_w = 1/(1-p)$. The flow is given by

$$J_{\text{sep}} = (1-\rho)(1-p). \tag{6}$$

From the above analysis, we find that the vehicles in the state of braking between $0 < \rho < \rho_2$ decrease and the capacity of the road approaches more closely the empirical data than that predicted by the NaSch model due to the role of the stochastic delay prior to deterministic deceleration while the increase of the braking vehicles between $\rho_1 < \rho < \rho_2$ due to the role of the stochastic delay and deterministic deceleration at the same time will frequently lead to the breakdown of flow and traffic jam. Therefore, the exchange of the order of the stochastic delay and deterministic deceleration has indeed a remarkable effect on traffic flow.

B. Three-phase traffic model and analysis

More recently, Huang analytically studied the three-phase traffic model [15] using the cellular automaton approach. But

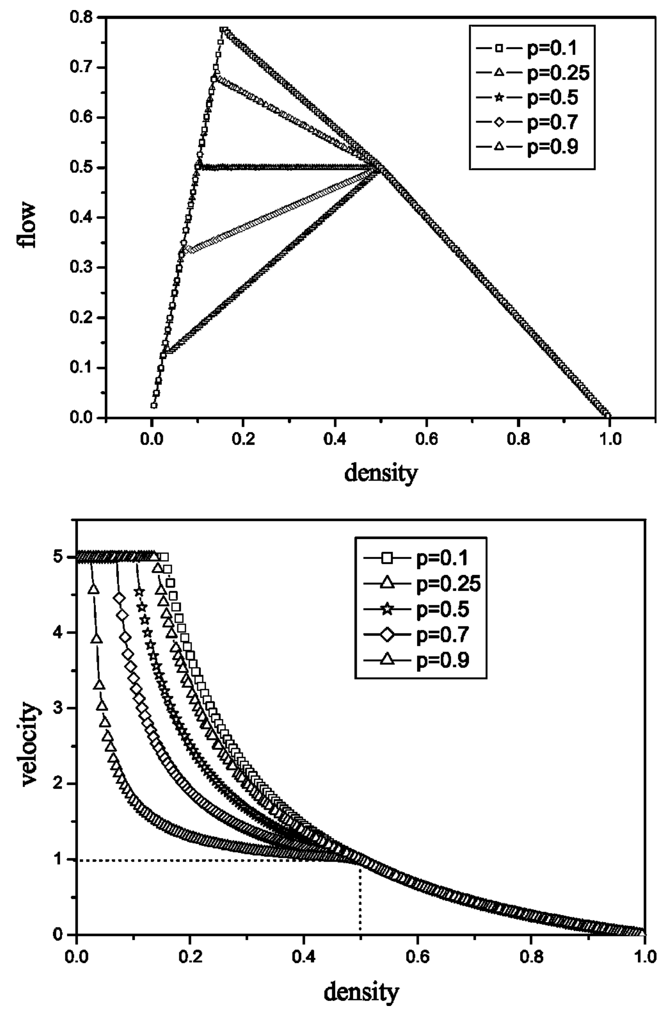

FIG. 4. (a) Fundamental diagram via numerical simulation with the same conditions as those of the NaSch model ($v_{\max}$=5, L=5×10^3) in the different situation of the delay probability p. (b) is a plot of the velocity-density relation corresponding to (a).

we find that we cannot reproduce the fundamental diagram Fig. 1 in his paper using his update rules because his model does not keep the conservation of number of vehicles. This may be a clerical error. But it is well known that the update rules in the theory of a cellular automaton is very important to determine the evolution of vehicles. We consider that step (iii) in his model causes a collision between successive vehicles. Thus, we only change the order of rule in the NaSch model by letting step (iii) go first and obtain the following model, referred to as the noise-first model.

(i) Noise,

$$v_n \to \max(v_n - 1, 0) \text{ with probability } p.$$

This rule reflects the fact that some drivers first consider decelerating with the probability p at random. In fact, the noise affects the process of acceleration.

(ii) Acceleration,

$$v_n \to \min(v_n + 1, v_{\max}).$$

This rule describes the case in which all drivers want to drive fast.

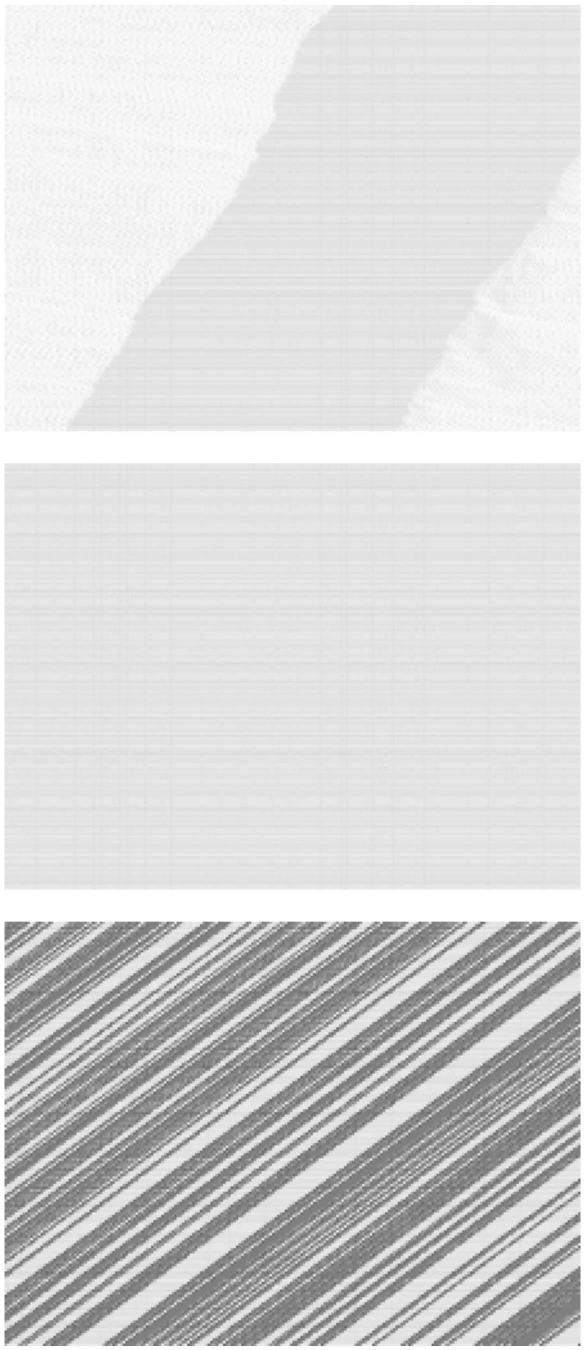

FIG. 5. The space-time plot describes the evolution of traffic flow in the cases of the different density. (a) $\rho=0.2$, (b) $\rho=0.5$, (c) $\rho=0.7$.

(iii) Braking,

$$v_n \rightarrow \min(v_n, \mathrm{gap}_n).$$

This rule represents the deterministic braking behaviors to avoid accidents.

(iv) Update of positions,

$$x_n(\mathrm{t}+1) \rightarrow \mathrm{x}_n + v_n.$$

This rule indicates each vehicle is moving forward according to its new velocity determined in steps (i)–(iii).

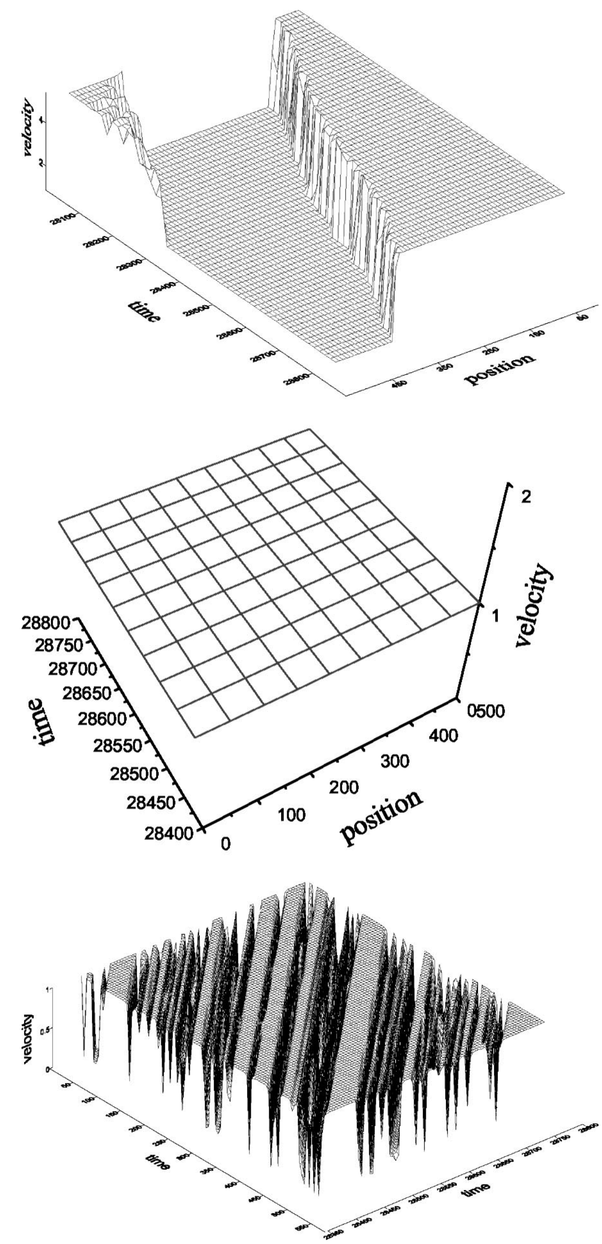

FIG. 6. The plots of the velocity distribution correspond to Fig. 5 in the cases of the different density. (a) $\rho=0.2$, (b) $\rho=0.5$, (c) $\rho=0.7$.

We obtain a fundamental diagram with our model under the same simulation conditions as those of the NaSch model, as shown in Fig. 4(a), and the corresponding velocity-density curve is given in Fig. 4(b). It can be found that there are three different parts—$0<\rho<\rho_1$, $\rho_1<\rho<\rho_2$, and $\rho_2<\rho<1$—which correspond to three different phases, i.e., free flow, low-speed flow, and jam. Interestingly, as the density ρ approaches 0.5, the velocity in low-speed flow becomes equal and all are equal to 1. The relation of the velocity density is as follows:

$$v = \begin{cases} v_{\max} & \rho < \rho_1 = \frac{1-p}{v_{\max}+1-2p} \\ \frac{1}{2\rho} - \frac{(1-2\rho)(2p-1)}{\rho} & \frac{1-p}{v_{\max}+1-2p} < \rho < \rho_2 = \frac{1}{2} \\ \frac{1}{\rho} - 1 & \rho \geqslant \frac{1}{2} \end{cases} .$$

The space-time plots (Fig. 5) and the plots of the velocity distribution (Fig. 6) verify low-speed flow, uniform low-speed flow, and jam for the cases of the different densities, respectively. This model is different from the NaSch model and has three different phases, i.e., free flow, low-speed flow, and jam. Even if the density of traffic flow surpasses the critical point, the jam can disappear. From the space-time plots in Fig. 5, we find that there is a collective region with directive motion at the same velocity of 1. From the fundamental diagram it can be seen that, as the density approaches 0.5, all vehicles form uniform flow with the identical velocity 1 and the situation has no relation to the delay probability p. It shows the basic characteristics of synchronized flow [16]. As the density becomes larger than $\rho=0.5$, vehicles will congest and stop-and-go traffic will occur.

III. Conclusions

In this paper, we study the effects of the orders of the evolutive rule on traffic flow. It has been found that the CA traffic model is sensitive to the orders of the evolutive rule by simulation. By changing the evolutive steps, we obtain two different traffic models. We analyze the mechanism of two different traffic models and corresponding traffic behaviors in detail and compare them with the NaSch model. The first model considers the effects of noise on the deterministic acceleration. It is a model to describe sensitive behaviors of the rabbit drivers through the rearrangement of stochastic delay and deterministic deceleration. This model can adapt even more to reflect the complicated traffic behaviors of real traffic, such as the metastable state, separation of phases, and hysteresis. The fundamental diagram obtained by numerical simulation shows the capacity of the road to approach more closely the empirical data compared with those of the NaSch model, in which the noise plays an important role. In the second model, the noise prior to deterministic acceleration is taken into account. This model has three different phases, i.e., free flow, low-speed flow, and jam. Moreover, even if the density of traffic flow surpasses the critical point and after the flow reaches the transit capacity, the jam can disappear. As the density reaches 0.5, the flow becomes uniform with identical velocity 1, which shows the characteristics of synchronized flow. Furthermore, the results indicate that the noise has indeed a remarkable effect on traffic flow.

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